Investigation of Scalar Field Metrics and Respective Visualization Techniques

Christopher P. Kappe*, Michael Böttinger†, Heike Leitte‡

Background and Objectives

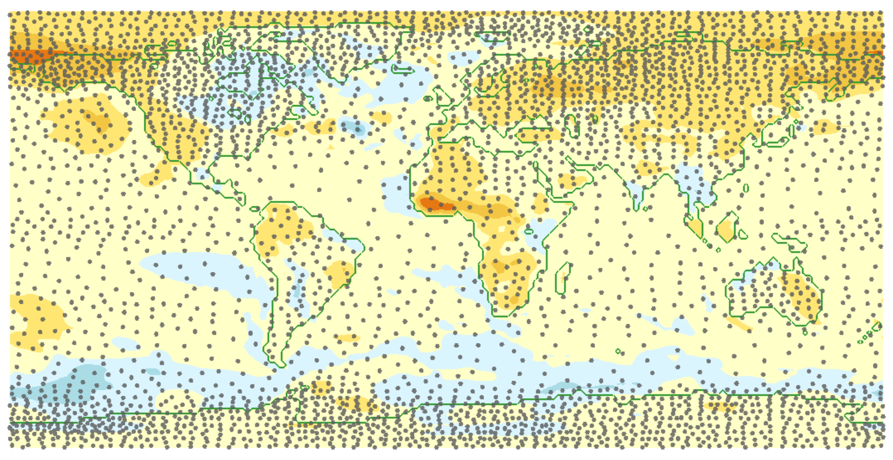
- Visualization of sets of scalar fields (e.g. time series or simulation ensembles).
- Computational comparison of scalar fields.

Mathematical Model

- Discrete scalar fields can be viewed as nD points where n is the number of grid points.
- Similarity or distance functions (metrics) can be (and have been) defined for such points.

Ensemble Summary View

E.g. mean by color and standard deviation by dot density.

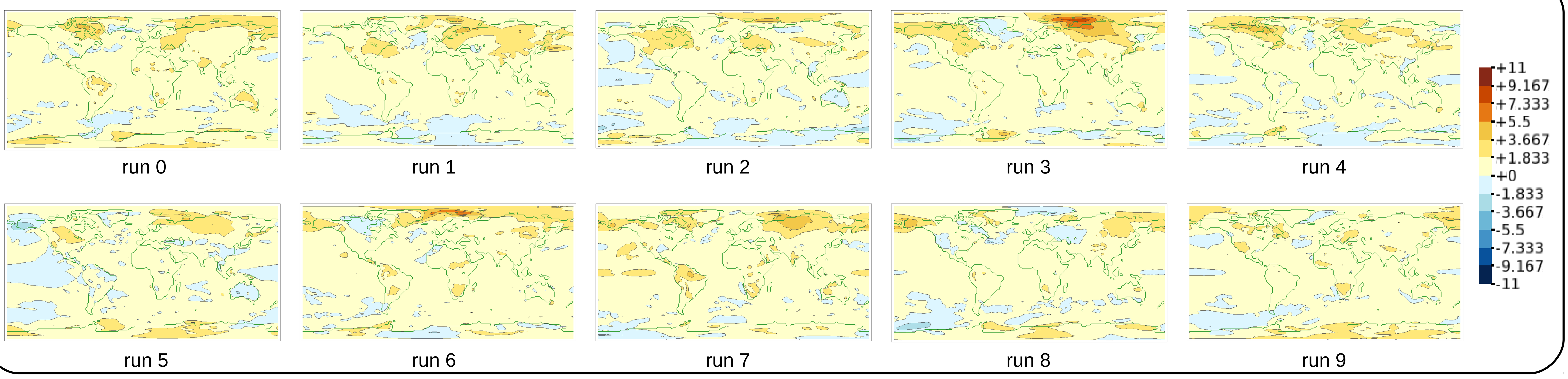


A Few Well-established Measures

Manhattan Distance: the sum of the magnitudes of the differences of all the scalars. $d(a,b)=\sum_{i=1}^{n}|a_i-b_i|$

Mean Squared Error: mean of the squared differences of all the scalars. $d(a,b)=\frac{1}{n}\sum_{i=1}^{n}|a_i-b_i|^2$

Cosine Distance: one minus the cosine (in [-1,1]) between two scalar fields in nD space.
$d(a,b)=1-\cos(\sphericalangle(a,b))=1-\frac{a\cdot b}{\|a\|\|b\|}$



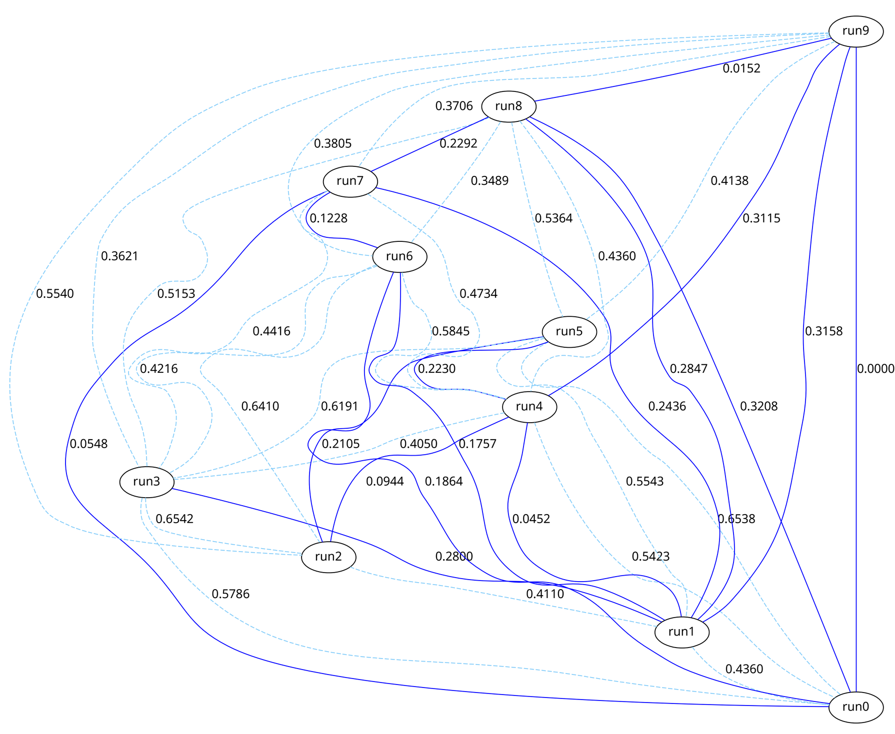
On the right one can see a distance matrix for the data set shown above. The lower left part contains the values for the cosine distance and the upper right shows the values for the Manhattan distance. In both cases the values have been normalized from [min,max] to [0,1]. A blue-white-red colormap has been applied to the values so that extreme values can be spotted more easily.

run	0	1	2	3	4	5	6	7	8	9
0		0.4360	0.8104	0.5786	0.5423	0.6538	0.1864	0.0548	0.3208	0.0000
1	0.3191		0.4110	0.2800	0.0452	0.5543	0.1757	0.2436	0.2847	0.3158
2	0.6058	0.4057		0.6542	0.0944	0.2105	0.6410	1.0000	0.8382	0.5540
3	0.3044	0.2052	0.4271		0.4050	0.6191	0.4216	0.4416	0.5153	0.3621
4	0.4449	0.1072	0.2846	0.2922		0.2230	0.5845	0.4734	0.4360	0.3115
5	0.4866	0.5215	0.4304	0.3544	0.4622		0.8465	0.8019	0.5364	0.4138
6	0.1998	0.1212	0.4804	0.2480	0.5140	0.7201		0.1228	0.3489	0.3805
7	0.0409	0.1339	0.8384	0.1333	0.3914	0.6640	0.1320		0.2292	0.3706
8	0.3280	0.4718	1.0000	0.5212	0.7575	0.7631	0.4808	0.1847		0.0152
9	0.0000	0.3221	0.5282	0.2816	0.4303	0.3826	0.3474	0.2309	0.1228	

Neighborhood Graph

for the Manhatten distance.

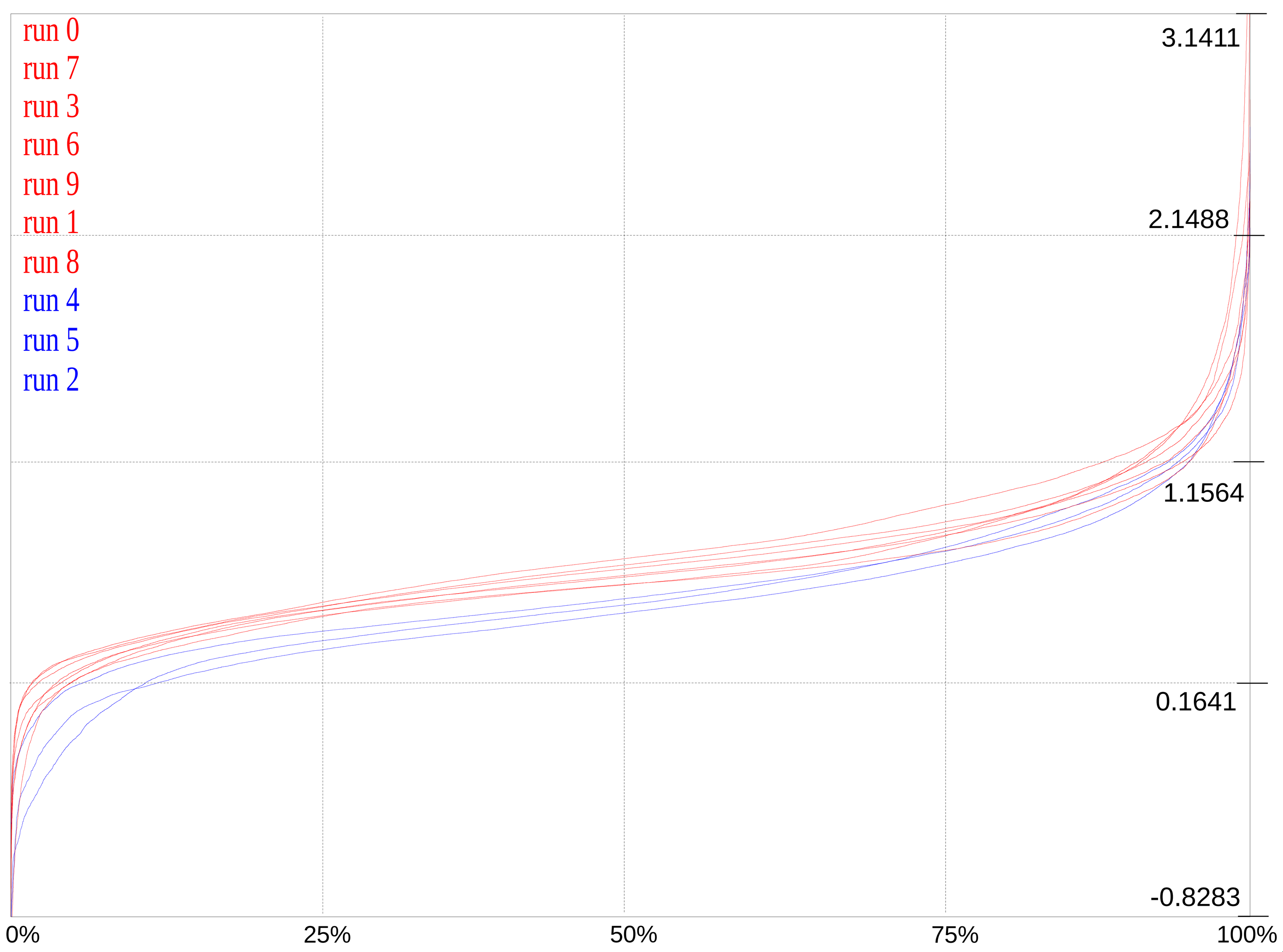
- d < ⅓ ⇒ dark blue edge
- d < ⅔ ⇒ light blue edge
- else: no edge



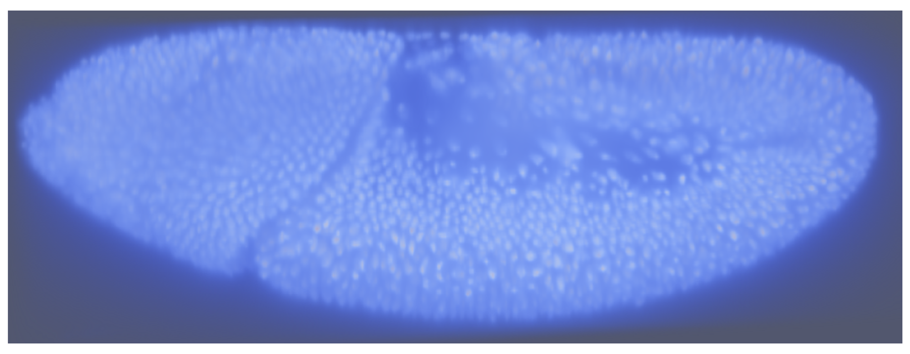
Scalar Field Signature

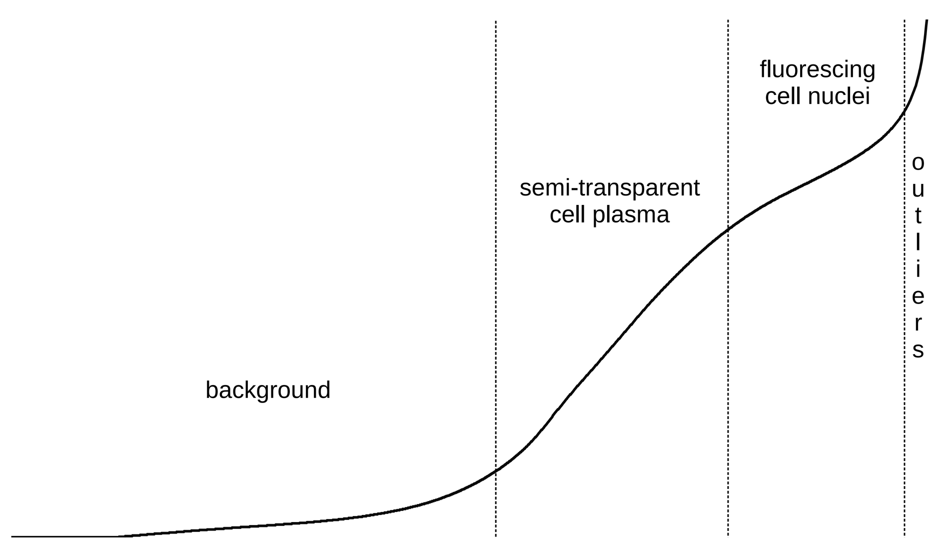
What we call a scalar field signature for a single scalar field given by an array of scalars x is obtained by sorting the elements of x in ascending order resulting in a new array y. This can be interpreted as samples of a monotonically nondecreasing function from the set of grid point indices {1,…,n} to the original range of scalars.

- Works for any kD domain.
- Works for any number of discrete data points.
- x-axis: some linearization of the original domain.
- y-axis: original image space.
- The range of values can be read.
- How often which value (bin of values) occurs can be read (approximately).
- Multiple scalar field signatures can be plotted in the same figure (e.g. for an ensemble).
- Can easily be animated (e.g. for time series).
- Certain similarity measures can be estimated.
- Used for a set of scalar fields, it can be enriched with the signature of the mean.
- The shape of the curve represents statistical characteristics of the field (e.g. roughly a Gaussian distribution of the scalars as it is the case in this figure).



3D Example





TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

*) kappe@cs.uni-kl.de
†) boettinger@dkrz.de
‡) leitte@cs.uni-kl.de

DKRZ DEUTSCHES KLIMARECHENZENTRUM