Convergence of the spatial schemes in the COSMO model and requirements on higher order spatial convergence

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1. Motivation

The reliability of climate models has to be based on the theoretical consistency of the equations of motion and the ability of the models to reproduce the observed weather and climate. An important aspect of the models is, which of the qualities of the analytical model shall and can be preserved with the discretised model.

The limited area models and an increasing number of global circulation models are using a discretisation in the physical space rather than in the spectral space. The widely used 2nd order central difference scheme exhibits a phase and amplitude error originating in a weak representation of the high wave numbers. The kinetic energy of the not represented fraction of the resolved flow field exceeds usually the kinetic energy of the unresolved scales. Scale selective (i.e. with no phase and amplitude error at resolved scales) and conservative (with respect to mass, energy and vertex conserved quantity) discretisation methods are known for simplified models. The increasing computing power makes their operational application more and more feasible.

First steps towards higher order schemes for non-hydrostatic models like WRF and COSMO have been done and their convergence properties have been demonstrated for idealised advection tests. The results however have not to be valid for the full model system. The COSMO schemes are examined here in the operational model system analysing the 2D mountain flow. The requirements of the such a test are discussed.

2. Introduction

Discretisations of the advection operator for non-hydrostatic models

- 2nd to 6th order advection schemes have been implemented at DWD in the 3rd order Runge Kutta Dynamical core of the COSMO model by appying the higher order derivative. They are identical with the schemes suggested by Wicker and Skamarock (2002) for the 1D advection test with constant u.

$$
\begin{aligned}
f_j^{(1)}(q) &:= -u\frac{q_j - q_{j-1}}{\Delta x}\\
f_j^{(2)}(q) &:= -u\frac{q_{j+1} - q_{j-1}}{2\,\Delta x}\\
f_j^{(3)}(q) &:= -u\frac{2q_{j+1} + 3q_j - 6q_{j-1} + q_{j-2}}{6\,\Delta x}\\
f_j^{(4)}(q) &:= -u\frac{-(q_{j+2} - q_{j-2}) + 8(q_{j+1} - q_{j-1})}{12\,\Delta x}\\
f_j^{(5)}(q) &:= -u\frac{-3q_{j+2} + 30q_{j+1} + 20q_j - 60q_{j-1} + 15q_{j-2} - 2q_{j-3}}{60\,\Delta x}\\
f_j^{(6)}(q) &:= -u\frac{(q_{j+3} - q_{j-3}) - 9(q_{j+2} - q_{j-2}) + 45(q_{j+1} - q_{j-1})}{60\,\Delta x}
\end{aligned}
$$

Further development of the schemes implemented

- On a staggered mesh the fields have to be interpolated. The higher order interpolation was implemented for the advection terms.

Centered differences higher order advection

As shown by Morinishi et al. (1998), JCP 143, 90-124 for incompressible flows, energy conserving higher order advection requires a higher order discretisation and higher or-der advection. Both have to be energy conserving. The following schemes are suggested:

2nd order derivative

$$\delta_{n\lambda}\psi = \frac{\psi(\lambda + n\,h_1/2, \phi, z) - \psi(\lambda - n\,h_1/2, \phi, z)}{n\,h_1} \qquad \text{1stD}$$

$$\delta_{n\lambda}\psi = \partial_\lambda\psi + \sum_{i=1}^{\infty}\frac{n^{2i}\,\partial_\lambda^{2i+1}\psi}{(2i+1)!\,2^{2i}}\,h^{2i} \qquad \text{Taylor exp. of RHS of 1stD}$$

2nd order interpolation

$$\overline{\psi}^{n\lambda} = \frac{\psi(\lambda + nh_1/2, \phi, z) + \psi(\lambda - nh_1/2, \phi, z)}{2} = \frac{\psi_{i+n/2,j,k} + \psi_{i-n/2,j,k}}{2} \qquad \text{INT}$$

$$\overline{\psi}^{n\lambda} = \psi + \sum_{i=1}^{\infty}\frac{n^{2i}\,\partial_\lambda^{2i}\psi}{2i!\,2^{2i}}\,h^{2i} \qquad \text{Taylor exp. of RHS of INT}$$

Higher order advection on a staggered grid

$$\overline{\psi}^{On,\lambda} = \sum_n a_n\overline{\psi}^{n\lambda} \qquad \delta_\lambda^{On}\psi = \sum_n a_n\delta_{n\lambda}\psi \quad \text{with} \quad n\varepsilon\{n_1, \cdots, n_{m/2}\}$$

$$\boxed{(\mathrm{v}\cdot\nabla\mathrm{u})^{\mathrm{n}} = \mathrm{u}\delta_\lambda^{\mathrm{n}}\mathrm{u} + \overline{\overline{\mathrm{v}}^{\mathrm{On},\phi}}^{\mathrm{On},\lambda}\delta_\phi^{\mathrm{On}}\mathrm{u} + \overline{\overline{\mathrm{w}}^{\mathrm{On,z}}}^{\mathrm{On},\lambda}\delta_{\mathrm{z}}^{\mathrm{On}}\mathrm{u}}$$

4th order: $a_2 = 4/3, \quad a_4 = -1/3$

$a_1 = 9/8, \quad a_3 = -1/8$

2D hydrostatic mountain flow idealized test case for the COSMO model: Configurations

The stationary mountain flow, hydrostatic linear case (Baldauf, 2009) has been investigated to show the convergence properties of the horizontal discretisations of the advection operator suggested by Wicker and Skamarock and 4th order Morinishi et al. (1998).

Initial conditions: w_0=0, u_0=10m/s, **Domain size:** Lx=50 a, 1.6 a < Lz < 3 a, t = 360 t0
Typical quantities: u_0=10m/s, t_0= a/u_0=1000s , a=10km (mountain width)

The spatial convergence properties of the advection schemes have been investigated systematically for different configurations aiming to exclude the influence of other error sources. The configurations used to obtain the results shown in section 3 are listed in the following:

PLT012 (COSMO schemes): Δ t=2.5 s (CFL = 0.025 at Δx=1km) , e-folding time of Rayleigh-damping at the upper boundary: tau= 200s, starting at rdheight=11km, domain height zmax=19,5 km, number of vertical levels ke=195, stretched grid, Δx=Δy : 0.125, 0.25, 0.5, 1.0, 2.0 and 4.0 km

PLT067 (Morinishi-scheme): as PLT012 but 4th order interpolation for 4th order advection scheme and improved configuration: exp(-10 x/x0) for lateral boundary relaxation, optimized Rayleigh damping coefficient (tau=400), CFL=0.05, zmax=25km, ke=500, rdheight=13.700m

PLT029 (CFL-dependence): as PLT012 but for dx=2km, tau=200 (nrdtau varying from 40 to 160) and constant vertical grid stretching dz=100m and zmax=30km and rdheight=137000m

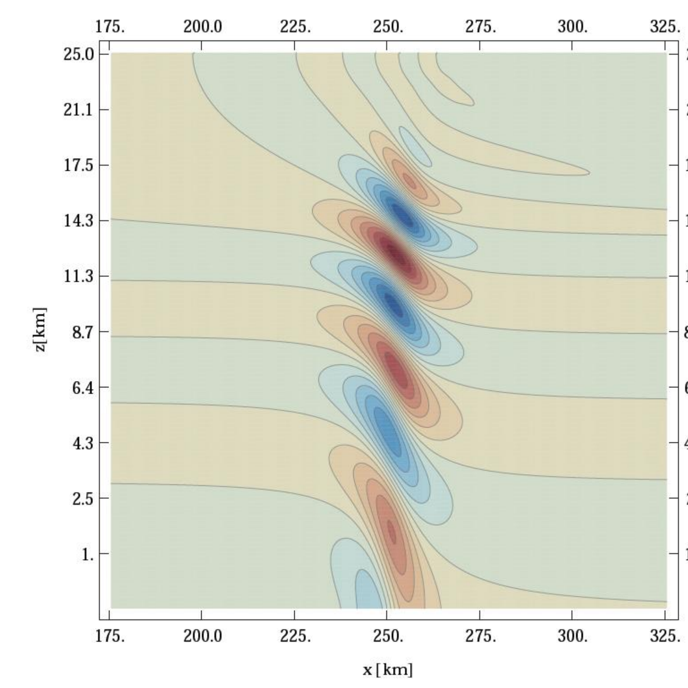
PLT058 (tau-dependence) : as PLT029 but CFL=0.05

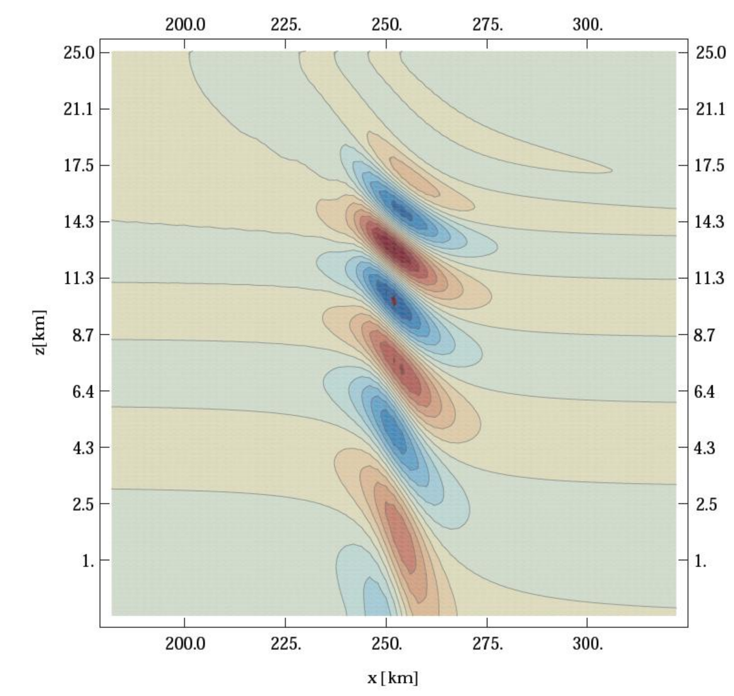
- Classical error norms L0, L1, and L2 for horizontal and vertical wind components were calculated.

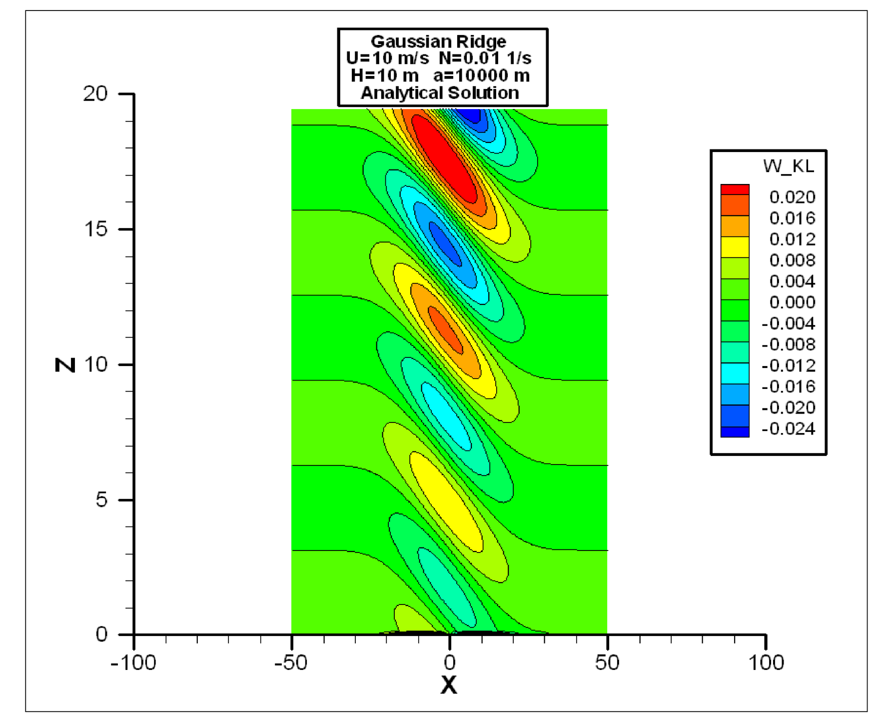
- A configuration has been developed in order to meet (1) the assumptions of the test case and (2) to keep the discretisation errors in time and the vertical discretisation (not tested) smaller than the horizontal discretisation error considered. In particular the boundary conditions had to be improved.

3. Results

Linear hydrostatic regime (Klemp-Lilly (1978))





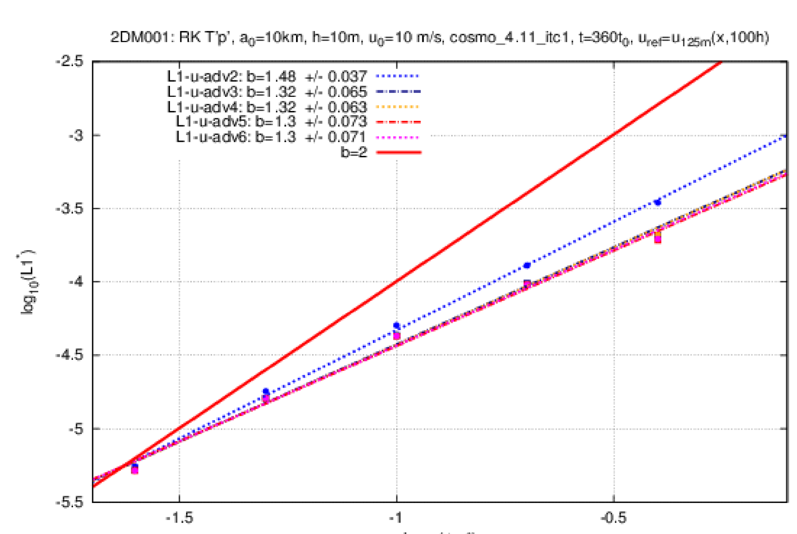


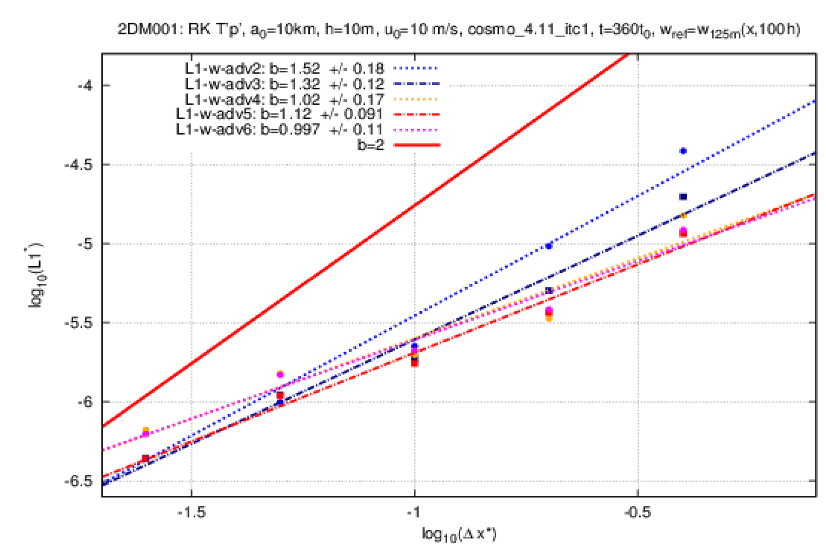
dx = 500m dx = 2000m

COSMO-solution, PLT067

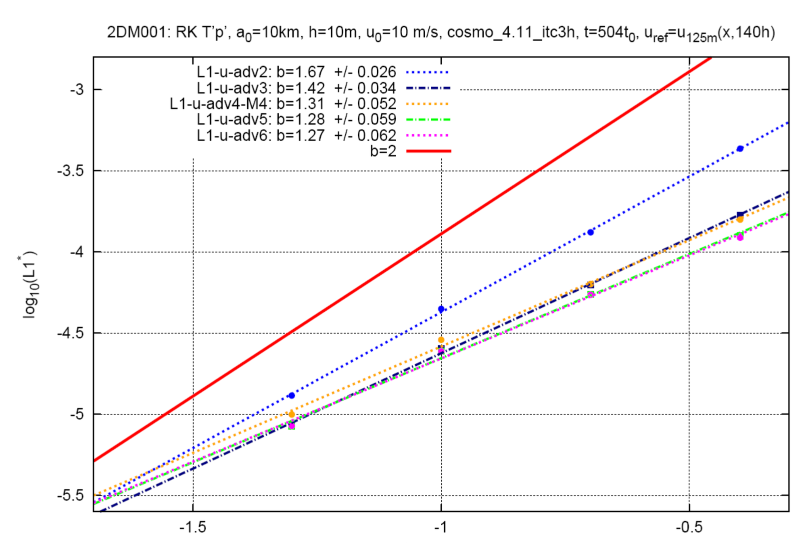
Analytical solution (Klemp-Lilly (1978) *JAS*)

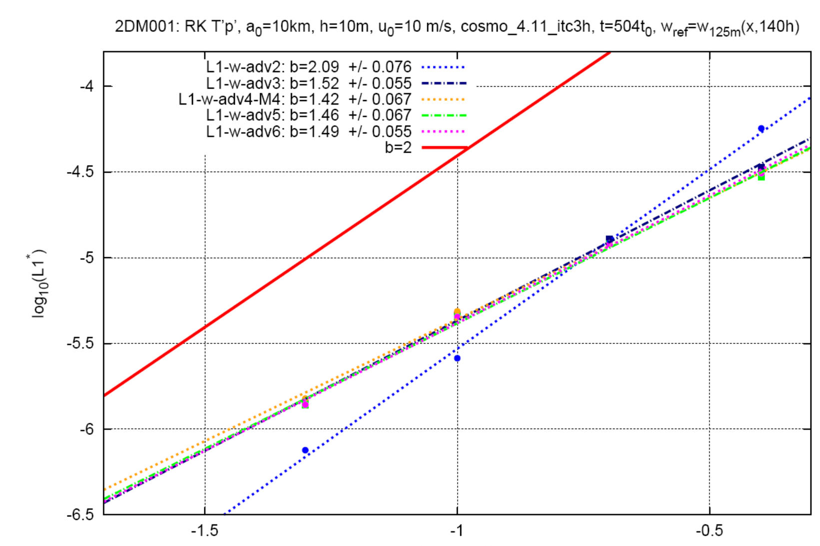
Convergence properties of error norms for u and w



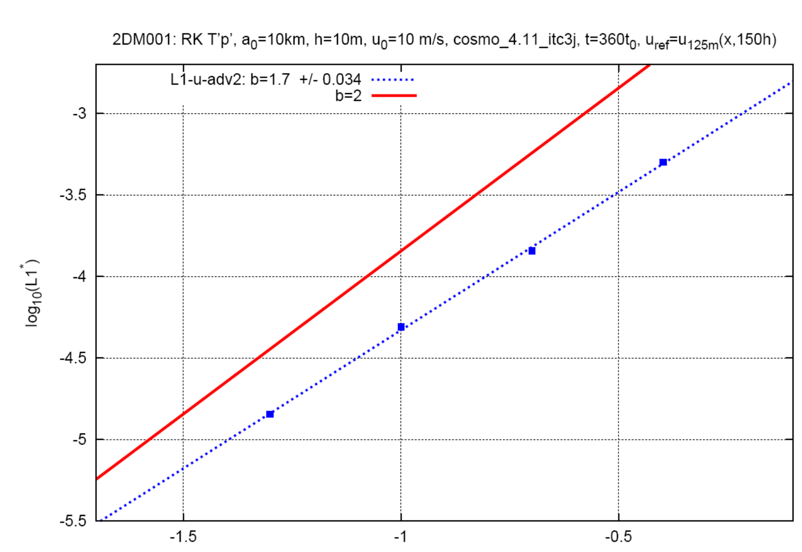


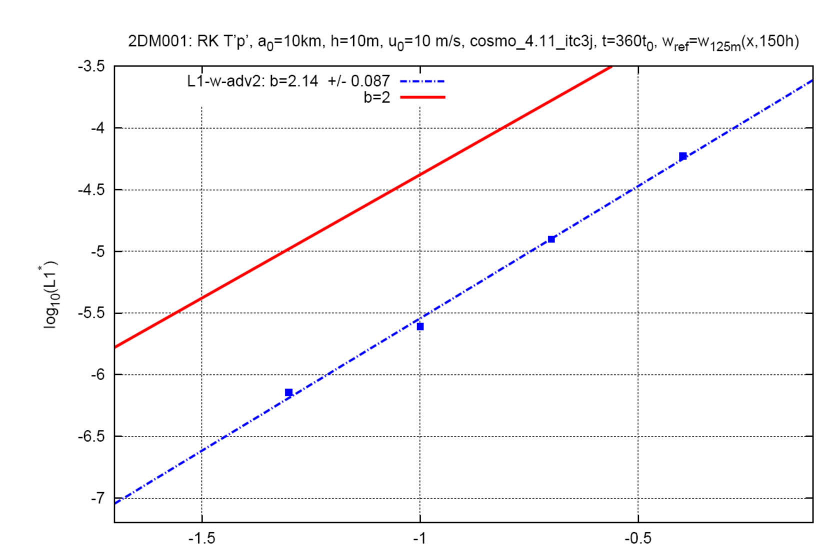
COSMO schemes (PLT012): a) L1 norm for u **b) L1 norm for w**



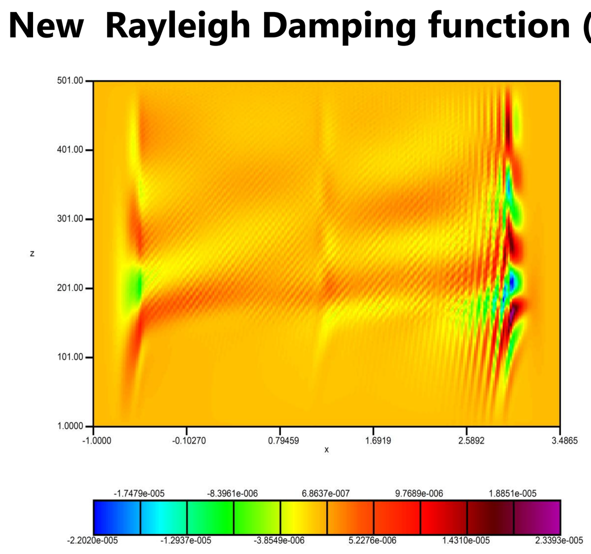


COSMO schemes (PLT067): c) L1 norm of u **d) L1 norm of w**
with 4th order Morinishi

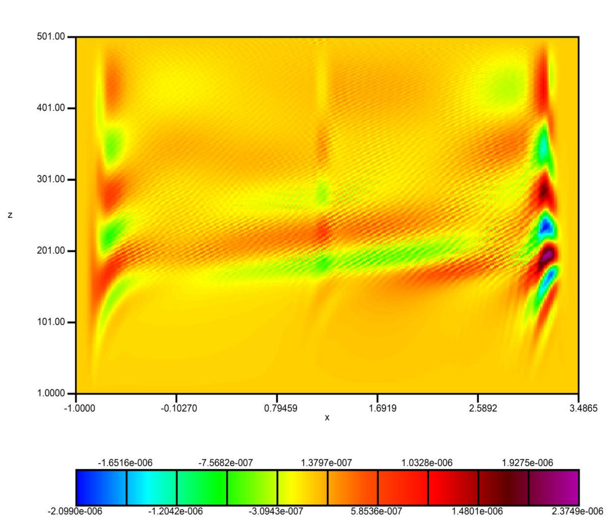




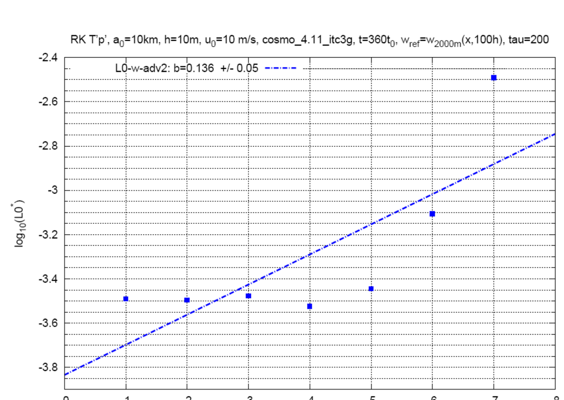
New Rayleigh Damping function (PTT014): f) L1 norm u **g) L1 norm of w**



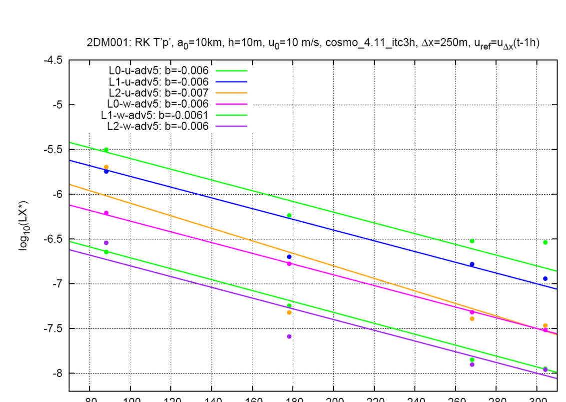
h) Cross-sections of 1hr instationarity of w: (with default boundary relaxation properties)



i) Cross-section of 1hr instationarity of w (with optimized boundary relaxation properites)



j) tau-dependence L0 norm of w for (PLT058)



k) Instationarity of the norms for the 5adv scheme. 3.6 t*=1h

Figure (a) and (b) show the convergence properties of the L1(u) and L1(w) norm for the 2nd to 6th order COSMO advection schemes derived from the 2D mountain flow test case using the PLT012 configuration (see section 2).

Figures (c) and (d): as (a) and (b) but for the improved configura-tion PLT067 and with 4th order Morinishi instead of 4th order COSMO. The 250m solutions have been used as reference at t=40h=144 To. The simulations are restarted from the 100h solution of PLT012.

Figures (e) and (f): as (c) and (d) but for the new Rayleigh damping function (1+cos(Pi x/xref))**4.

Figrue (g) shows the new Rayleigh damping function in comparison to the standard (cos(Pi x/xref))**2 function.

Figure (h) exhibits the convergence with respect to the time step dt at 2km horizontal resolution . The reference of the L0 error norm is the dt=0,625s solution.

Figure (i) shows the convergence of the norms with the simulation time.

Figure (j) exhibits the dependence on the Rayleigh damping at the top of the atmosphere for tau(N)=100 N. The error norms L0(N) are calculated as differences for tau(N+1) and tau(N).

Figure (k) exhibits the convergence of the instationarity for the norms for the 5 adv scheme. For the instationarity norm the time t-1h is used as reference.

4. Summary and outlook

The results of the PLT012 configuration selected at the beginning of the study are inconsistent with the theoretical solution. An improved configuration (PLT067) exhibits a result consistent with the theory for the 2nd order scheme. The comparison between PLT067 and PLT012 exhibits that the configuration has a substantial influence on the convergence results and that the accuracy of the convergence is not trustable.
The orders of convergence of the error norms calculated is 2 for the 2nd order scheme and below two for all higher order schemes. The convergence properties for different norms and variables are still inconsistent.

A detailed analysis of the results has shown a significant impact of the damping terms at the lateral and at the upper boundary on the results since the convergence tests require a high accuracy, not needed for real case studies. Figures (h) shows the results from the simulations with the default relaxation coefficients function at the lateral boundary revealing disturbances at the boundary and figure (i) reveals the simulations with an optimized lateral boundary condition.
Correct results can be expected as soon as the results for the high resolution reference simulation (125m) for all numerical schemes will be available and the new damping function is now used at the lateral boundaries too.

The results show the high accuracy requirements of convergence tests and the need for a careful analysis of the results obtained. A consistency of different error norms (here L0, L1 and L2) for different variables (here u and w) is indespensable.

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5. References

1) Baldauf, M. (2009). COSMO Priority Project "Further Developments of the Runge-Kutta Time Integration Scheme" (RK): Final Report. Technical Report 14, COSMO. 2) Wicker, L. J., and W. C. Skamarock, 2002: Time-Splitting methods for Elastic Models Using Forward Time Schemes. *Mon. Wea. Rev.*, **130,** 2088-2097. 3) Morinishi, Y., Lund, T., Vasilyev, O., and Moin, P. (1998). Fully Conservative Higher Order Finite Difference Schemes for Incompressible Flow. JoCP , 143, 90–124